# Cost- Benefit Analysis of Two-Dissimilar Units Warm Standby System Subject to Electromagnetic Vibrations with Switch Failure 

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#### Abstract

In this paper, we present a two-unit dissimilar warm standby systems subject to electromagnetic vibrations(denoted as EM vibrations) with switch failure .The EM vibrations and failure rates are constant whereas the repair time distributions are taken to be arbitrary. The EM vibrations are non-instantaneous and cannot occur simultaneously in both the units and when there are EM vibrations within specified limit of a unit, it operates as normal as before but if these are beyond the specified limit the operation of the unit stop automatically so that excessive damage of the unit is avoided and the EM vibrations goes on, some characteristics of the stopped unit change which we call failure of the unit. We have calculated MTSF, Availability ,the expected busy time of the server for repairing the failed unit under EM vibration in (0,t], the expected busy time of the server for repair of dissimilar units by the repairman in( $0, t]$, the expected busy time of the server for repair of switch in (0,t], the expected number of visits by the repairman for repairing the different units in (0,t], the expected number of visits by the repairman for repairing the switch in (0,t] and cost analysis. Special case by taking repair time distribution as exponential are discussed and graphs are drawn.


Keyword- dissimilar units, warm standby, switch failure, EM vibrations

## 1. INTRODUCTION

We present a two-unit dissimilar warm standby systems subject to EM vibrations with switch failure .The EM vibrations and failure rates are constant where as the repair time distributions are taken to be arbitrary. The EM vibrations are non-instantaneous and cannot occur simultaneously in both the units and when there are EM vibrations within specified limit of a unit, it operates as normal as before but if these are beyond the specified limit the operation of the unit stop automatically so that excessive damage of the unit is avoided and when the EM vibrations goes on, some characteristics of the stopped unit change which we call failure of the unit.

For example, when a satellite launched into its orbit around the earth there is a region of electromagnetic field. When the satellite passes through such field some equipment present in the
satellite might be disturbed due to electromagnetic vibrations in the space which may deviate the satellite from the orbit causing it directionless for a while. To control this situation it is possible with the help of sensors that for some time the working of the equipment under the influence of electromagnetic vibrations may stop and the sensors again detect where and when electromagnetic field finished after which in the satellite, through the sensor control unit, the working of the equipment under influence of electromagnetic vibrations starts immediately. It is assumed that all the sensors system is perfectly working whenever needed.

## 2. ASSUMPTIONS

1. The system consists of two dissimilar warm standby units. The EM vibration and failure time of units and switch failure distributions are exponential with rates $\lambda_{1}, \lambda_{2}, \lambda_{3}$ and $\lambda_{4}$ respectively whereas the repairing rates for repairing the failed system due to EM vibrations and due to switch failure are arbitrary with $\mathrm{CDF} \mathrm{G}_{1}(\mathrm{t}) \& \mathrm{G}_{2}(\mathrm{t})$ respectively.
2. The operation of units stops automatically when EM vibrations occurs so that excessive damage of the unit can be prevented.
3. The EM vibrations actually failed the units. The EM vibrations are non-instantaneous and it cannot occur simultaneously in both the units.
4. The repair facility works on the come first serve (FCFS) basis.
5. The switches are imperfect and instantaneous.
6. All random variables are mutually independent.

## Symbols for states of the System

Superscripts O, WS, SO, F, SFO
Operative, Warm Standby, Stops the operation, Failed, Switch failed but operable respectively
Subscripts nv, uv, ur, wr, uR

No EM vibration, under EM vibration, under repair, waiting for repair, under repair continued respectively

Up states - 0,1,2,9 ; Down states - 3,4,5,6,7,8,10,11

## States of the System

$\mathbf{0}\left(\mathrm{O}_{\mathrm{nv}}, \mathrm{WS}_{\mathrm{nv}}\right)$

One unit is operative and the other unit is warm standby and there are no EM vibrations in both the units.

## $\mathbf{1}\left(\mathrm{SO}_{\mathrm{nv}}, \mathrm{O}_{\mathrm{nv}}\right)$

The operation of the first unit stops automatically due to EM vibrations and warm standby units starts operating.

## $\mathbf{2}\left(\mathrm{F}_{\mathrm{ur}}, \mathrm{O}_{\mathrm{nv}}\right)$

The first unit fails and undergoes repair after the EM vibrations are over and the second unit continues to be operative due to EM vibrations in it .

## 3( $\left.\mathrm{F}_{\mathrm{uR}}, \mathrm{SO}_{\mathrm{uv}}\right)$

The repair of the first unit is continued from state 2 and in the second unit stops automatically due to EM vibrations.

## 4( $\left.\mathrm{F}_{\mathrm{ur}}, \mathrm{SO}_{\mathrm{uv}}\right)$

The first unit fails and undergoes repair after the vibrations are over and the other unit also stops automatically due to EM vibrations.

## $\mathbf{5}\left(\mathrm{F}_{\mathrm{uR}}, \mathrm{F}_{\mathrm{wr}}\right)$

The repair of the first unit is continued from state 4 and the other unit is failed due to EM vibrations in it \& is waiting for repair.

## $\mathbf{6}\left(\mathrm{O}_{\mathrm{nv}}, \mathrm{F}_{\mathrm{ur}}\right)$

The repair of the first unit is completed \& it starts operation and the second unit which was waiting for repair undergoes repair.

## 7( $\mathrm{SO}_{\mathrm{uv}}, \mathrm{SFO}_{\mathrm{nv}, \mathrm{ur}}$ )

The operation of the first unit stops automatically due to EM vibrations from state 0 and during switchover to the second unit switch fails and undergoes repair.

## 8( $\mathrm{F}_{\mathrm{wr}}, \mathbf{S F O}_{\mathrm{nv}, \mathrm{ur}}$ )

The repair of the switch is continued from state 7 and the first unit fails after EM vibrations and is waiting for repair.

## 9( $\mathrm{O}_{\mathrm{nv}}, \mathrm{SO}_{\mathrm{uv}}$ )

The first unit is operative and the warm standby dissimilar unit comes under the EM vibrations.


Fig. 1 The State Transition Diagram


## $\mathbf{1 0}\left(\mathrm{SO}_{\mathrm{nv}}, \mathrm{F}_{\mathrm{ur}}\right)$

The operation of the first unit stops automatically due to EM vibrations and the second unit fails and undergoes repair after the EM vibrations are over.

## $\mathbf{1 1}\left(\mathbf{F}_{\mathrm{wr}}, \mathrm{F}_{\mathrm{uR}}\right)$

The repair of the second unit is continued from state 10 and the first unit is failed and waiting for repair.

## Transition Probabilities

Simple probabilistic considerations yield the following expressions :
$\mathrm{p}_{01}=\frac{\lambda 1}{\lambda 1+\lambda 2+\lambda 4} \quad, \quad \mathrm{P}_{07}=\frac{\lambda 2}{\lambda 1+\lambda 2+\lambda 4}$
$\mathrm{p}_{09}=\frac{\lambda 4}{\lambda 1+\lambda 2+\lambda 4} \quad, \mathrm{p}_{12}=\frac{\lambda 1}{\lambda 1+\lambda 3}, \mathrm{p}_{14}=\frac{\lambda 3}{\lambda 1+\lambda 3}$
$\left.\mathrm{P}_{20}=\mathrm{G}_{1}{ }^{*}\left(\lambda_{1}\right), \mathrm{P}_{22}{ }^{(3)}=\mathrm{G}_{1} \overline{(\lambda}_{1}\right), \mathrm{P}_{72}=\mathrm{G}_{2}{ }^{*}\left(\lambda_{4}\right), \mathrm{P}_{72}{ }^{(8)}=\mathrm{G}_{2}{ }^{*}\left(\lambda_{4}\right)=\mathrm{P}_{78}-$

Also other values can be defined.

We can easily verify that
$\mathrm{P}_{01}+\mathrm{P}_{07}+\mathrm{P}_{09}=1, \mathrm{P}_{20}+\mathrm{P}_{22}{ }^{(3)}=1, \mathrm{P}_{22}{ }^{(3)}=1$,
$\mathrm{P}_{60}=1, \mathrm{P}_{72}+\mathrm{P}_{72}{ }^{(8)}+\mathrm{P}_{74}=1, \mathrm{P}_{9,10}=1, \mathrm{P}_{10,2}+\mathrm{P}_{10,2}{ }^{(11)}=1$

And mean sojourn time are
$\mu_{0}=\mathrm{E}(\mathrm{T})=\int_{0}^{\infty} P[T>t] d t$

## Mean Time To System Failure

We can regard the failed state as absorbing

$$
\begin{align*}
& \quad \theta_{0}(t)=Q_{01}(t)[s] \theta_{1}(t)+Q_{09}(t)[s] \theta_{9}(t)+Q_{07}(t) \\
& \theta_{1}(t)=Q_{12}(t)[s] \theta_{2}(t)+Q_{14}(t), \theta_{2}(t)=Q_{20}(t)[s] \theta_{0}(t)+Q_{22}^{(3)}(t) \\
& \theta_{4}(t)=Q_{9,10}(t) \tag{3-5}
\end{align*}
$$

Taking Laplace-Stiltjes transform of eq. (3-5) and solving for
$Q_{0}^{*}(s)=\mathrm{N}_{1}(\mathrm{~s}) / \mathrm{D}_{1}(\mathrm{~s})$
Where
$\mathrm{N}_{1}(\mathrm{~s})=Q_{01}^{*}(s) \quad\left\{Q_{12}^{*}(s) Q_{22}^{(3) *}(s)+Q_{14}^{*}(s)\right\}+Q_{09}^{*}(s) Q_{9,10}^{*}(s)+Q_{07}^{*}(s)$
$\mathrm{D}_{1}(\mathrm{~s})=1-Q_{01}^{*}(s) \quad Q_{12}^{*}(s) Q_{20}^{*}(s)$
Making use of relations (1) \& (2) it can be shown that $Q_{0}^{*}(0)=1$, which implies
that $\theta_{1}(t)$ is a proper distribution.
$\left.\operatorname{MTSF}=\mathrm{E}[\mathrm{T}]=\frac{d}{d s} Q_{01}^{*}(s) \right\rvert\,=\left(\mathrm{D}_{1}^{\prime}(0)-\mathrm{N}_{1}{ }^{\prime}(0)\right) / \mathrm{D}_{1}(0)$
$\mathrm{S}=0$
$=\left(\mu_{0}+\mathrm{p}_{01} \mu_{1}+\mathrm{p}_{01} \mathrm{p}_{12} \mu_{2}+\mathrm{p}_{09} \mu_{9}\right) /\left(1-\mathrm{p}_{01} \mathrm{p}_{12} \mathrm{p}_{20}\right)$
where
$\mu_{0}=\mu_{01}+\mu_{07}+\mu_{09}, \mu_{1}=\mu_{12}+\mu_{14}, \mu_{2}=\mu_{20}+\mu_{22}{ }^{(3)}, \mu_{9}=\mu_{9,10}$

## Availability analysis

Let $\mathrm{M}_{\mathrm{i}}(\mathrm{t})$ be the probability of the system having started from state I is up at time t without making any other regenerative state belonging to E . By probabilistic arguments, we have

The value of $\mathrm{M}_{0}(\mathrm{t}), \mathrm{M}_{1}(\mathrm{t}), \mathrm{M}_{2}(\mathrm{t}), \mathrm{M}_{4}(\mathrm{t})$ can be found easily.
The point wise availability $A_{i}(t)$ have the following recursive relations
$\mathrm{A}_{0}(\mathrm{t})=\mathrm{M}_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{9}(\mathrm{t})$
$\mathrm{A}_{1}(\mathrm{t})=\mathrm{M}_{1}(\mathrm{t})+\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{4}(\mathrm{t}), \mathrm{A}_{2}(\mathrm{t})=\mathrm{M}_{2}(\mathrm{t})+\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})$
$\mathrm{A}_{4}(\mathrm{t})=\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{6}(\mathrm{t}), \mathrm{A}_{6}(\mathrm{t})=\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{0}(\mathrm{t})$
$\mathrm{A}_{7}(\mathrm{t})=\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{4}(\mathrm{t})$
$\mathrm{A}_{9}(\mathrm{t})=\mathrm{M}_{9}(\mathrm{t})+\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{10}(\mathrm{t}), \mathrm{A}_{10}(\mathrm{t})=\mathrm{q}_{10,2}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})(7-14)$
Taking Laplace Transform of eq. (7-14) and solving for $\hat{A}_{0}(s)$

$$
\begin{equation*}
\hat{A}_{0}(s)=\mathrm{N}_{2}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s}) \tag{15}
\end{equation*}
$$

Where
$\mathrm{N}_{2}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{\widehat{M}_{0}(\mathrm{~s})+\hat{q}_{01}(\mathrm{~s}) \widehat{M}_{1}(\mathrm{~s})+\hat{q}_{09}(\mathrm{~s}) \widehat{M}_{9}(\mathrm{~s})\right\}+\widehat{M}_{2}(\mathrm{~s})\left\{\hat{q}_{01}(\mathrm{~s}) \hat{q}_{42}(\mathrm{~s})+\right.$
$\left.\widehat{q}_{07}(\mathrm{~s})\left(\hat{q}_{72}(\mathrm{~s})+\hat{q}_{73}{ }^{(8)}(\mathrm{s})\right)+\hat{q}_{09}(\mathrm{~s}) \hat{q}_{9,10}(\mathrm{~s})\left(\hat{q}_{10,2}(\mathrm{~s})+\hat{q}_{10,2}{ }^{(11)}(\mathrm{s})\right)\right\}$
$\mathrm{D}_{2}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{1-\hat{q}_{46}{ }^{(5)}(\mathrm{s}) \hat{q}_{60}(\mathrm{~s})\left(\hat{q}_{01}(\mathrm{~s}) \hat{q}_{44}(\mathrm{~s})+\hat{q}_{07}(\mathrm{~s}) \hat{q}_{74}(\mathrm{~s})\right)\right.$
$-\widehat{q}_{20}(\mathrm{~s})\left\{\hat{q}_{01}(\mathrm{~s}) \hat{q}_{12}(\mathrm{~s})+\hat{q}_{07}(\mathrm{~s})\left(\hat{q}_{72}(\mathrm{~s})\right)+\hat{q}_{72}{ }^{(8)}(\mathrm{s})+\hat{q}_{09}(\mathrm{~s}) \hat{q}_{9,10}(\mathrm{~s})\right.$
$\left.\left(\hat{q}_{10,2}(\mathrm{~s})+\hat{q}_{10,2}{ }^{(11)}(\mathrm{s})\right)\right\}$
The steady state availability
$\mathrm{A}_{0}=\lim _{t \rightarrow \infty}\left[A_{0}(t)\right]=\lim _{s \rightarrow 0}\left[s \hat{A}_{0}(s)\right]=\lim _{s \rightarrow 0} \frac{s N_{2}(s)}{D_{2}(s)}$
Using L' Hospitals rule, we get
$\mathrm{A}_{0}=\lim _{s \rightarrow 0} \frac{N_{2}(s)+s N_{2} \prime(s)}{D_{2^{\prime}}(s)}=\frac{N_{2}(0)}{D_{2^{\prime}}(0)}$
Where
$\mathrm{N}_{2}(0)=\mathrm{p}_{20}\left(\widehat{M}_{0}(0)+\mathrm{p}_{01} \widehat{M}_{1}(0)+\mathrm{p}_{09} \widehat{M}_{9}(0)\right)+\widehat{M}_{2}(0)\left(\mathrm{p}_{01} \mathrm{p}_{12}+\mathrm{p}_{07}\left(\mathrm{p}_{72}\right.\right.$
$\left.\left.+\mathrm{p}_{72}{ }^{(8)}+\mathrm{p}_{09}\right)\right)$
$\mathrm{D}_{2}{ }^{\prime}(0)=\mathrm{p}_{20}\left\{\mu_{0}+\mathrm{p}_{01} \mu_{1}+\left(\mathrm{p}_{01} \mathrm{p}_{14}+\mathrm{p}_{07} \mathrm{p}_{74}\right) \mu_{4}+\mathrm{p}_{07} \mu_{7}+\mathrm{p}_{07} \mu_{7}+\mathrm{p}_{09}\left(\mu_{9}+\mu_{10}\right)\right.$
$+\mu_{2}\left\{1-\left(\left(\mathrm{p}_{01} \mathrm{p}_{14}+\mathrm{p}_{07} \mathrm{p}_{74}\right)\right\}\right.$
$\mu_{4}=\mu_{46}^{(5)}, \mu_{7}=\mu_{72}+\mu_{72}^{(8)}+\mu_{74}, \mu_{10}=\mu_{10,2}+\mu_{10,2}^{(11)}$
The expected up time of the system in $(0, t]$ is
$\lambda_{u}(\mathrm{t})=\int_{0}^{\infty} A_{0}(z) d z$ So that $\widehat{\lambda_{u}}(\mathrm{~s})=\frac{\widehat{\mathrm{A}}_{0}(\mathrm{~s})}{\mathrm{s}}=\frac{N_{2}(S)}{S D_{2}(S)}$
The expected down time of the system in $(0, t]$ is
$\lambda_{d}(\mathrm{t})=\mathrm{t}-\lambda_{u}(\mathrm{t})$ So that $\widehat{\lambda_{d}}(\mathrm{~s})=\frac{1}{\mathrm{~s}^{2}}-\widehat{\lambda_{u}}(\mathrm{~s})$
The expected busy period of the server for repairing the failed unit under EM vibration in (0,t]
$\mathrm{R}_{0}(\mathrm{t})=\mathrm{S}_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{9}(\mathrm{t})$
$\mathrm{R}_{1}(\mathrm{t})=\mathrm{S}_{1}(\mathrm{t})+\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{4}(\mathrm{t}), \mathrm{R}_{2}(\mathrm{t})=\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})$
$\mathrm{R}_{4}(\mathrm{t})=\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{6}(\mathrm{t}), \mathrm{R}_{6}(\mathrm{t})=\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{0}(\mathrm{t})$
$\mathrm{R}_{7}(\mathrm{t})=\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{4}(\mathrm{t})$
$\mathrm{R}_{9}(\mathrm{t})=\mathrm{S}_{9}(\mathrm{t})+\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{10}(\mathrm{t}), \quad \mathrm{R}_{10}(\mathrm{t})=\mathrm{q}_{10,2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})$
Taking Laplace Transform of eq. (19-26) and solving for $\widehat{R_{0}}(s)$

$$
\begin{equation*}
\widehat{R_{0}}(s)=\mathrm{N}_{3}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s}) \tag{27}
\end{equation*}
$$

Where
$\mathrm{N}_{2}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{\hat{S}_{0}(\mathrm{~s})+\hat{q}_{01}(\mathrm{~s}) \hat{S}_{1}(\mathrm{~s})+\hat{q}_{09}(\mathrm{~s}) \hat{S}_{9}(\mathrm{~s})\right\}$ and $\mathrm{D}_{2}(\mathrm{~s})$ is already defined.
In the long run, $\quad \mathrm{R}_{0}=\frac{N_{3}(0)}{D_{2}(0)}$
where $\mathrm{N}_{3}(0)=\mathrm{p}_{20}\left(\hat{S}_{0}(0)+\mathrm{p}_{01} \hat{S}_{1}(0)+\mathrm{p}_{09} \hat{S}_{9}(0)\right)$ and $\mathrm{D}_{2}{ }^{\prime}(0)$ is already defined.
The expected period of the system under EM vibration in $(0, \mathrm{t}]$ is
$\lambda_{r v}(\mathrm{t})=\int_{0}^{\alpha} R_{0}(z) d z$ So that $\widehat{\lambda_{r v}}(\mathrm{~s})=\frac{\widehat{\mathrm{R}}_{0}(\mathrm{~s})}{\mathrm{s}}$
The expected Busy period of the server for repair of dissimilar units by the repairman in $(0, t]$
$\mathrm{B}_{0}(\mathrm{t})=\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{9}(\mathrm{t})$
$\mathrm{B}_{1}(\mathrm{t})=\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{4}(\mathrm{t}), \mathrm{B}_{2}(\mathrm{t})=\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})$
$\mathrm{B}_{4}(\mathrm{t})=\mathrm{T}_{4}(\mathrm{t})+\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{6}(\mathrm{t}), \mathrm{B}_{6}(\mathrm{t})=\mathrm{T}_{6}(\mathrm{t})+\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{0}(\mathrm{t})$
$\mathrm{B}_{7}(\mathrm{t})=\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{4}(\mathrm{t})$
$\mathrm{B}_{9}(\mathrm{t})=\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{10}(\mathrm{t}), \mathrm{B}_{10}(\mathrm{t})=\mathrm{T}_{10}(\mathrm{t})+\left(\mathrm{q}_{10,2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t}) \quad(29-36)\right.$
Taking
Laplace Transform of eq. (29-36) and solving for $\widehat{B_{0}}(s)$
$\widehat{B_{0}}(s)=\mathrm{N}_{4}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s})$

Where
$\mathrm{N}_{4}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{\hat{q}_{01}(\mathrm{~s}) \hat{q}_{14}(\mathrm{~s})\left(\widehat{T}_{4}(\mathrm{~s})+\hat{q}_{46}{ }^{(5)}(\mathrm{s}) \widehat{T}_{6}(\mathrm{~s})\right)+\hat{q}_{07}{ }^{(3)}(\mathrm{s}) \widehat{q}_{74}(\mathrm{~s})\left(\widehat{T}_{4}(\mathrm{~s})\right.\right.$ $\left.\left.+\hat{q}_{46}{ }^{(5)}(\mathrm{s}) \widehat{T}_{6}(\mathrm{~s})\right)+\hat{q}_{09}(\mathrm{~s}) \widehat{q}_{09,10}(\mathrm{~s}) \widehat{T}_{10}(\mathrm{~s})\right)$
And $\mathrm{D}_{2}(\mathrm{~s})$ is already defined.

In steady state, $\mathrm{B}_{0}=\frac{N_{4}(0)}{D_{2^{\prime}}(0)}$
where $\mathrm{N}_{4}(0)=\mathrm{p}_{20}\left\{\left(\mathrm{p}_{01} \mathrm{p}_{14}+\mathrm{p}_{07} \mathrm{p}_{74}\right)\left(\widehat{T}_{4}(0)+\widehat{T}_{6}(0)\right)+\mathrm{p}_{09} \widehat{T}_{10}(0)\right\}$ and $\mathrm{D}_{2}^{\prime}(0)$ is already defined.
The expected busy period of the server for repair in $(0, t]$ is
$\lambda_{r u}(\mathrm{t})=\int_{0}^{\infty} B_{0}(z) d z$ So that $\widehat{\lambda_{r u}}(\mathrm{~s})=\frac{\widehat{\mathrm{B}}_{0}(\mathrm{~s})}{\mathrm{s}}$

## The expected Busy period of the server for repair of switch in ( $\mathbf{o}, \mathrm{t}$ ]

$\mathrm{P}_{0}(\mathrm{t})=\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{9}(\mathrm{t})$
$\mathrm{P}_{1}(\mathrm{t})=\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{4}(\mathrm{t}), \mathrm{P}_{2}(\mathrm{t})=\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})$
$\mathrm{P}_{4}(\mathrm{t})=\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{6}(\mathrm{t}), \mathrm{P}_{6}(\mathrm{t})=\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{0}(\mathrm{t})$
$\mathrm{P}_{7}(\mathrm{t})=\mathrm{L}_{7}(\mathrm{t})+\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{4}(\mathrm{t})$
$\mathrm{P}_{9}(\mathrm{t})=\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{10}(\mathrm{t}), \mathrm{P}_{10}(\mathrm{t})=\left(\mathrm{q}_{10,2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})(40-47)$
Taking Laplace Transform of eq. (40-47) and solving for

$$
\begin{equation*}
\widehat{P_{0}}(s)=\mathrm{N}_{5}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s}) \tag{48}
\end{equation*}
$$

where $\mathrm{N}_{2}(\mathrm{~s})=\hat{q}_{07}(\mathrm{~s}) \hat{L}_{7}(\mathrm{~s})\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)$ and $\mathrm{D}_{2}(\mathrm{~s})$ is defined earlier.
In the long run, $\mathrm{P}_{0}=\frac{N_{5}(0)}{D_{2}^{\prime}(0)}$
where $\mathrm{N}_{5}(0)=\mathrm{p}_{20} \mathrm{p}_{07} \hat{L}_{4}(0)$ and $\mathrm{D}_{2}{ }^{\prime}(0)$ is already defined.
The expected busy period of the server for repair of the switch in $(0, t]$ is
$\lambda_{r s}(\mathrm{t})=\int_{0}^{\alpha} P_{0}(z) d z$ So that $\widehat{\lambda_{r s}}(\mathrm{~s})=\frac{\widehat{\mathrm{P}}_{0}(\mathrm{~s})}{\mathrm{s}}$
The expected number of visits by the repairman for repairing the different units in $(0, t]$
$\mathrm{H}_{0}(\mathrm{t})=\mathrm{Q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{1}(\mathrm{t})+\mathrm{Q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{7}(\mathrm{t})+\mathrm{Q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{9}(\mathrm{t})$
$\mathrm{H}_{1}(\mathrm{t})=\mathrm{Q}_{12}(\mathrm{t})[\mathrm{c}]\left[1+\mathrm{H}_{2}(\mathrm{t})\right]+\mathrm{Q}_{14}(\mathrm{t})[\mathrm{c}]\left[1+\mathrm{H}_{4}(\mathrm{t})\right], \mathrm{H}_{2}(\mathrm{t})=\mathrm{Q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{0}(\mathrm{t})+\mathrm{Q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{2}(\mathrm{t})$
$\mathrm{H}_{4}(\mathrm{t})=\mathrm{Q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{6}(\mathrm{t}), \mathrm{H}_{6}(\mathrm{t})=\mathrm{Q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{0}(\mathrm{t})$
$\mathrm{H}_{7}(\mathrm{t})=\left(\mathrm{Q}_{72}(\mathrm{t})+\mathrm{Q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{H}_{2}(\mathrm{t})+\mathrm{Q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{4}(\mathrm{t})$
$\mathrm{H}_{9}(\mathrm{t})=\mathrm{Q}_{9,10}(\mathrm{t})[\mathrm{c}]\left[1+\mathrm{H}_{10}(\mathrm{t})\right], \mathrm{H}_{10}(\mathrm{t})=\left(\mathrm{Q}_{10,2}(\mathrm{t})[\mathrm{c}]+\mathrm{Q}_{10,2}{ }^{(11)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{H}_{2}(\mathrm{t})(51-58)$
Taking Laplace Transform of eq. (51-58) and solving for $H_{0}^{*}(s)$
$H_{0}^{*}(s)=\mathrm{N}_{6}(\mathrm{~s}) / \mathrm{D}_{3}(\mathrm{~s})$

Where
$\mathrm{N}_{6}(\mathrm{~s})=\left(1-Q_{22}{ }^{(3)}(\mathrm{s})\right)\left\{Q^{*}{ }_{01}(\mathrm{~s})\left(Q^{*}{ }_{12}(\mathrm{~s})+Q^{*}{ }_{14}(\mathrm{~s})\right)+Q^{*}{ }_{09}(\mathrm{~s}) Q^{*}{ }_{9,10}(\mathrm{~s})\right\}$
$\mathrm{D}_{3}(\mathrm{~s})=\left(1-Q_{22}{ }^{(3)^{*}}(\mathrm{~s})\right)\left\{1-\left(Q^{*}{ }_{01}(\mathrm{~s}) Q^{*}{ }_{14}(\mathrm{~s})+Q^{*}{ }_{07}(\mathrm{~s}) Q^{*}{ }_{74}(\mathrm{~s})\right) Q_{46}{ }^{(5)^{*}}(\mathrm{~s}) Q^{*}{ }_{60}(\mathrm{~s})\right\}$

- $Q^{*}{ }_{20}(\mathrm{~s})\left\{Q^{*}{ }_{01}(\mathrm{~s}) Q^{*}{ }_{12}(\mathrm{~s})+Q^{*}{ }_{07}(\mathrm{~s})\left(Q^{*}{ }_{72}(\mathrm{~s})\right)+Q^{*}{ }_{72}{ }^{(8)}(\mathrm{s})+\right.$
$Q^{*}{ }_{09}(\mathrm{~s}) Q^{*}{ }_{9,10}(\mathrm{~s})\left(Q^{*}{ }_{10,2}(\mathrm{~s})+\mathrm{Q}_{\left.\left.10,{ }^{(11)}{ }^{*}(\mathrm{~s})\right)\right\}}\right.$
In the long run, $\mathrm{H}_{0}=\frac{N_{6}(0)}{D_{3}^{\prime}(0)}$
where $\mathrm{N}_{6}(0)=\mathrm{p}_{20}\left(\mathrm{p}_{01}+\mathrm{p}_{09}\right)$ and $\mathrm{D}^{\prime}{ }_{3}(0)$ is already defined.

The expected number of visits by the repairman for repairing the switch in (0,t]
$\mathrm{V}_{0}(\mathrm{t})=\mathrm{Q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{1}(\mathrm{t})+\mathrm{Q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{7}(\mathrm{t})+\mathrm{Q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{9}(\mathrm{t})$
$\mathrm{V}_{1}(\mathrm{t})=\mathrm{Q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})+\mathrm{Q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{4}(\mathrm{t}), \mathrm{V}_{2}(\mathrm{t})=\mathrm{Q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{0}(\mathrm{t})+\mathrm{Q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})$
$\mathrm{V}_{4}(\mathrm{t})=\mathrm{Q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{6}(\mathrm{t}), \mathrm{V}_{6}(\mathrm{t})=\mathrm{Q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{0}(\mathrm{t})$
$\mathrm{V}_{7}(\mathrm{t})=\left(\mathrm{Q}_{72}(\mathrm{t})\left[1+\mathrm{V}_{2}(\mathrm{t})\right]+\mathrm{Q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})+\mathrm{Q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{4}(\mathrm{t})$
$\mathrm{V}_{9}(\mathrm{t})=\mathrm{Q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{10}(\mathrm{t}), \mathrm{V}_{10}(\mathrm{t})=\left(\mathrm{Q}_{10,2}(\mathrm{t})+\mathrm{Q}_{10,2}{ }^{(11)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})$
Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $V_{0}{ }^{*}(s)$

$$
\begin{equation*}
V_{0}^{*}(s)=\mathrm{N}_{7}(\mathrm{~s}) / \mathrm{D}_{4}(\mathrm{~s}) \tag{69}
\end{equation*}
$$

where $\mathrm{N}_{7}(\mathrm{~s})=Q^{*}{ }_{07}(\mathrm{~s}) Q^{*}{ }_{72}(\mathrm{~s})\left(1-Q_{22}{ }^{(3)}(\mathrm{s})\right)$ and $\mathrm{D}_{4}(\mathrm{~s})$ is the same as $\mathrm{D}_{3}(\mathrm{~s})$
In the long run, $\mathrm{V}_{0}=\frac{N_{7}(0)}{D_{4}^{\prime}(0)}$
where $\mathrm{N}_{7}(0)=\mathrm{p}_{20} \mathrm{p}_{07} \mathrm{p}_{72}$ and $\mathrm{D}^{\prime}{ }_{3}(0)$ is already defined.

## Cost Benefit Analysis

The cost-benefit function of the system considering mean up-time, expected busy period of the system under vibrations when the units stops automatically, expected busy period of the server for repair of unit \& switch, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for switch failure.
The expected total cost-benefit incurred in $(0, \mathrm{t}]$ is
$\mathrm{C}(\mathrm{t})=$ Expected total revenue in $(0, \mathrm{t}]$ - expected total repair cost for switch in $(0, \mathrm{t}]$

- expected total repair cost for repairing the units in $(0, t$ ]
- expected busy period of the system under vibration when the units automatically stop in $(0, t]$
- expected number of visits by the repairman for repairing the switch in $(0, t]$
- expected number of visits by the repairman for repairing of the units in $(0, t]$

The expected total cost per unit time in steady state is
$\mathrm{C}=\lim _{t \rightarrow \infty}(C(t) / t)=\lim _{s \rightarrow 0}\left(s^{2} C(s)\right)$
$=\mathrm{K}_{1} \mathrm{~A}_{0}-\mathrm{K}_{2} \mathrm{P}_{0}-\mathrm{K}_{3} \mathrm{~B}_{0}-\mathrm{K}_{4} \mathrm{R}_{0}-\mathrm{K}_{5} \mathrm{~V}_{0}-\mathrm{K}_{6} \mathrm{H}_{0}$

Where
$\mathrm{K}_{1}$ - revenue per unit up-time,
$\mathrm{K}_{2}$ - cost per unit time for which the system is under switch repair
$\mathrm{K}_{3}$ - cost per unit time for which the system is under unit repair
$\mathrm{K}_{4}$ - cost per unit time for which the system is under EM vibrations when units automatically stop.
$\mathrm{K}_{5}$ - cost per visit by the repairman for which switch repair,
$\mathrm{K}_{6}$ - cost per visit by the repairman for units repair.

## 3. CONCLUSION

After studying the system, we have analysed graphically that when the failure rate, EM vibration rate increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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