

Hybrid Simplex Method for Economic Load Dispatch

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Abstract: The modern power system around the world has grown in complexity of interconnection and power demand. The focus has shifted towards enhanced performance, low cost, reliable and clean power. In this changed perspective, scarcity of energy resources and increasing power generation cost necessitates optimal economic load dispatch (ELD). The main objective of the economic load dispatch problem is to determine the optimal combination of power outputs of all generating units so as to meet the required demand at minimum cost while satisfying the constraints. Over the past decade, in order to solve economic load dispatch problem, many salient methods have been developed such as hierarchical numerical method, genetic algorithm, evolutionary programming, neural network approaches, differential evolution, particle swarm optimization, and the hybrid methods. In this paper hybrid Simplex method is applied to solve ELD problem, which is a local search method combined with random exploitation of the worst point. Modifications in the simplex method are made by adding random exploitation of the worst point and by using multiple simplexes instead of a single simplex. The promising result on the benchmark function shows the applicability of the method for solving ELD problem. The test results obtained for three and six generator system prove the authentication of the method.

Keywords: Economic load dispatch, hybrid simplex method, random exploitations, worst point, multiple simplexes.

1. INTRODUCTION

Economic dispatch is defined as the process of allocating generation levels to the committed generating units so that the system load may be supplied entirely and most economically. Practically the ELD problem is nonlinear, non-convex with multiple local optimal points due to the inclusion of valve point loading effect, multiple fuel options with diverse equality and inequality constraints. Over the past decade, in order to solve economic load dispatch problem, many salient methods have been developed such as hierarchical numerical method, genetic algorithm, evolutionary programming, neural network approaches, differential evolution, particle swarm optimization, and the hybrid methods. Here hybrid simplex method is used which is a local search method with modifications of random exploitation of the worst point and multiple simplexes instead of a single simplex. These modifications are introduced in the simplex method to search

around the boundary of the feasible region and to control the convergence speed of the method. After implementation of the proposed method on the ELD problems, the results found were robust.

2. REVIEW OF LITERATURE

There are several studies that have been contributed by many researchers and engineers regarding the power system problem, i.e. ELD problem. Fink [1] described the valve-point loading logic which is intended to meet at any time in the most economical fashion a generation commitment. Dhillon [4] gave the economic emission load dispatch (EELD) problem is a multiple non-commensurable objective problem that minimizes both cost and emission together. Hou, *et al.* [6] has applied versatile optimization algorithm called modified particle swarm optimization algorithm (MPSO) for solving the economic dispatch problem of power systems. Coelho, *et al.* [10] demonstrated the feasibility of employing modified Particle swarm optimization approaches for efficient solving of economic load dispatch problems with generator constraints. Each method has its own advantages and disadvantages: however Hybrid simplex method provides robust results.

3. PROBLEM FORMULATION

The ELD problem can be formulated as single objective and multi objective problem which are nonlinear and non-convex in nature. Single objective problem can be formulated as the ELD without valve point effect, ELD with valve point loading effect, ELD with valve point loading effect and multiple fuel option. Multi objective formulation includes combined emission economic dispatch, multi-area emission dispatch, power generation under different utilities, maximization of generated power and irrigation.

3.1 Objective Function

The primary objective of any ELD problem is to reduce the operational cost by fulfilling the load demand within limit of constraints. The various kinds of objective function formulation are given below.

3.1.1 Single Objective Economic Cost Function: Economic cost function can be represented as a quadratic fuel cost objective function.

$$\text{Minimize: } F(P_{gi}) = \sum_{i=1}^n F_i(P_{gi}) \quad (1)$$

$$F_i(P_{gi}) = a_i P_{gi}^2 + b_i P_{gi} + c_i \text{ (Rs/h)} \quad (2)$$

where a_i, b_i, c_i = operating cost coefficients.

P_{gi} = Real power generation and the decision variable.

n = Number of generating plants.

$F(P_{gi})$ = Operating fuel cost of the i th plant.

3.1.2 Economic Cost Function with Valve Point Loading:

The generating units with multiple valves in steam turbines are available. The opening and closing of these valves are helpful to maintain the active power balance. However it adds the ripples in the cost function. This makes the objective function highly nonlinear. The sinusoidal functions are added to the quadratic function [8].

$$F(P_{gi}) = \sum_{i=1}^n (a_i P_{gi}^2 + b_i P_{gi} + c_i) + |d_i * \sin\{e_i * (P_{gi}^{\min} - P_{gi})\}| \quad (3)$$

where: P_{gi}^{\min} is the Minimum loading limit below which it is uneconomical to operate the unit.

3.2 Economic Load Dispatch with Transmission Losses

The transmission losses may vary from 5 to 15 per cent of the total load. Therefore, it is essential to account for transmission losses while developing the economic load dispatch policy. The ELD optimization problem is defined as [8]:

$$\text{Minimize: } F(P_{gi}) = \sum_{i=1}^n F_i(P_{gi}) \text{ Rs/h} \quad (4)$$

Subjected to:

The energy balance equation

$$\sum_{i=1}^n P_{gi} = P_D + P_L \quad (5)$$

The inequality constraints are given

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \quad (i = 1, 2, \dots, n) \quad (6)$$

Where: P_D is real power demand and P_L is the transmission power loss.

The transmission loss expression is :

$$P_L = B_{00} + \sum_{i=1}^n B_{i0} P_{gi} + \sum_{i=1}^n \sum_{j=1}^n P_{gi} B_{ij} P_{gj} \text{ MW} \quad (7)$$

With P_{gi} and P_{gj} are the real power injections at the i th and the j th buses, respectively. B_{00}, B_{i0}, B_{ij} are the loss coefficients which are constant under certain assumed conditions.

For economic load dispatch without transmission losses, P_L is supposed to be zero in the above equations.

4. HYBRID SIMPLEX METHOD

In the hybrid simplex method the constraint violation and the objective function are used separately. Both the constraint violation and the objective function are optimized in such a way that the constraint violation precedes the objective function i.e. feasibility of X is more important than minimization of $f(X)$. In the simplex method, a simplex is spanned by multiple search points and the simplex shows the region in which optimal solution will exist. However, when the simplex method is applied to the constrained optimization problems, points round the boundary of the feasible region are skipped when the simplex is reduced. The hybrid simple method avoids such skipping of the points of the feasible region by adding random exploitation of the worst point and by using multiple simplexes instead of a single simplex [3].

The following modifications are introduced in simplex method for solving constrained optimization problems and thus this method is called as hybrid simplex method.

- The first modification is the application of the α constrained method. The ordinary comparisons are replaced by the α level comparisons.
- The second modification is the introduction of the random exploitation of the worst point.
- The third modification is the introduction of multiple simplexes. It is known that the simplex S sometimes loses affine independence and the hybrid simplex method cannot find optimal solutions as well. To avoid such situation multiple simplexes are used in the hybrid simplex method. In the initialization step, the $N(> n + 1)$ points are generated. As initial points. When the centroid is determined, a simplex is composed by selecting the $n + 1$ points from all the points except the worst point and the simplex is used as the centroid. The centroids are defined by the different simplexes. So, even if some simplexes lose affine independence, the other affine independent simplexes can lead the search point. Here in this paper, $n + 2$ search points have been taken because $n + 1$ points are used for simplex and one point is used for the worst point.

5. ALGORITHM FORMULATION

The following steps are being performed in solving economic load dispatch problem using hybrid simplex method

5.1 Simplex Formation

The distance (d) between the vertices of the simplex is calculated

The simplex points s_1 and s_2 are found and then the simplex is formed using the generalized form i.e.

$$U = [x^0, x^1, x^2, \dots, x^N] = \begin{bmatrix} 0 & s_1 & s_2 & s_3 & \dots & s_N \\ 0 & s_2 & s_1 & s_2 & \dots & s_1 \\ 0 & s_2 & s_2 & s_1 & \dots & s_1 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & s_2 & s_2 & \dots & s_2 & s_1 \end{bmatrix} \quad (N = n + 2) \quad (8)$$

The hybrid simplex is formed as:

$$S = [U + x_i^{\min}] \quad (i = 1, 2, \dots, n) \quad (9)$$

5.2 Objective Evaluation

The objective function is evaluated on these simplex points and the membership function is found out using

$$\mu_{gi} = \begin{cases} 1 & , \text{if } g_i \leq 0 \\ 1 - \frac{g_i}{b_i} & , \text{if } 0 \leq g_i \leq b_i \\ 0 & , \text{otherwise} \end{cases} \quad (10)$$

$$\mu_{hj} = \begin{cases} 1 - \frac{|h_j|}{b_j} & , |h_j| < b_j \\ 0 & , \text{otherwise} \end{cases} \quad (11)$$

where b_i and b_j are the proper positive fixed numbers. Here, minimization is used for the combination operator so, the satisfaction levels of $\mu_{gi}(x)$ and $\mu_{hj}(x)$ is combined using

$$\mu(x) = \min \{ \mu_{gi}(x), \mu_{hj}(x) \} \quad (12)$$

The sorting of the function values is carried out to find the lowest (i.e. best), highest (i.e. worst), second highest (i.e. second worst) points i.e. x^l, x^h, x^s . The value of the objective function on these points is calculated f^l, f^h, f^s and correspondingly membership functions are also calculated and arranged $\mu^l, \mu^e, \mu^h, \mu^s$.

5.3 Centroid

From the simplex formed choose $n + 1$ vertices from S excluding the worst point and then the centroid is calculated by

$$x^0 = \frac{1}{n+1} \sum_{i \neq h}^{n+2} s_i \quad (13)$$

5.4 Reflection Point

The reflection point is found by using

$$x^r = (1 + a)x^0 - ax^h \quad (14)$$

and correspondingly value of the objective function on this point and membership functions (f^r and μ^r) are calculated.

5.5 Expansion Point

Calculate the expansion point x^e

$$x^e = (1 + b)x^0 - bx^h \quad (15)$$

The function value at this point is found and membership function is also calculated (f^e and μ^e). If the function value at a point is better than the best point in the simplex then the reflection is considered to have taken the simplex to a good region in the search space. Thus, an expansion along the direction from the centroid to the reflected point is performed with the help of Eq. (15). The amount of expansion is controlled by the factor b .

5.6 Contraction Point

The contraction point is found by using

$$x^c = (1 + c)x^0 - cx^h \quad (16)$$

Then the correspondingly value of the objective function on this point and membership functions (f^c and μ^c) are calculated. If the function value at the reflected point is better than the worst and worse than the next to worst point in the simplex, a contraction is made with a positive c value.

If the function value at the reflected point is worse or equal to the function value at the worst point then the contraction is carried out with the negative c value and is shown in Eq. (17)

$$x^c = (1 - c)x^0 + cx^h \quad (17)$$

5.7 Random Exploitation of the Worst Point

If the generated random number is less than the random exploitation rate (p_m) then the random exploitation of the worst point is performed. In which a second random number is generated and is compared on the basis of the following equation

$$x_i^h = \begin{cases} x_i^l + R * (x_i^h - x_i^l), R < 0.5 \\ x_i^h - R * (x_i^h - x_i^l), \text{otherwise} \end{cases} \quad (i = 1, 2, \dots, n) \quad (18)$$

and the previous x^h value with the new x^h value is replaced.

5.8 α Level Comparison

The α level comparisons are defined by a lexicographic order in which $\mu(x)$ precedes $f(x)$, because the feasibility of x is more important than the minimization of $f(x)$. The $f_1(f_2)$ and $\mu_1(\mu_2)$ are the function value and the satisfaction level respectively, at a point $x_1(x_2)$. Then for any α satisfying $0 \leq \alpha \leq 1$, the α level comparisons $<_\alpha$ and \leq_α

Between (f_1, μ_1) and (f_2, μ_2) are defined as follows

$$(f_1, \mu_1) <_\alpha (f_2, \mu_2) \Leftrightarrow \begin{cases} f_1 < f_2, & \text{if } \mu_1, \mu_2 > \alpha \\ f_1 < f_2, & \text{if } \mu_1 = \mu_2 \\ \mu_1 > \mu_2, & \text{otherwise} \end{cases} \quad (19)$$

$$(f_1, \mu_1) \leq_\alpha (f_2, \mu_2) \Leftrightarrow \begin{cases} f_1 \leq f_2, & \text{if } \mu_1, \mu_2 \geq \alpha \\ f_1 \leq f_2, & \text{if } \mu_1 = \mu_2 \\ \mu_1 > \mu_2, & \text{otherwise} \end{cases} \quad (20)$$

The updation of α is performed by using Eq. (21)

$$\alpha(t) = \begin{cases} 0.5(\mu^{\max} + \bar{\mu}) & ; t = 0 \\ (1 - \beta)\alpha(t-1) + \beta & ; 0 < t < \frac{t_{\max}}{2} \text{ and } (t \bmod T_\alpha = 0) \\ \alpha(t-1) & ; 0 < t < \frac{t_{\max}}{2} \text{ and } (t \bmod T_\alpha \neq 0) \\ 1 & ; t > \frac{t_{\max}}{2} \end{cases} \quad (21)$$

and set $\alpha = \alpha(t)$

5.9 Stopping criterion

The iterations will stop when the simplex S is converged enough, such as when the function value at the worst point minus the function value at the best point becomes sufficiently small.

$$|f^h - f^l| \leq \varepsilon \quad (22)$$

where t is the number of iterations. T_{\max} is the maximum number of iterations. β is the parameter for controlling α level and ε is the tolerance limit.

6. RESULTS

The implementation of hybrid simplex based optimization technique is done on the test function, three generator and six generator electrical power systems. The results are extensively

influenced by number of iterations. The test system which undergoes hybrid simplex optimization gives better performance evaluation.

6.1 Standard Test Function

$$\text{Minimize: } f(x) = x_1^2 + (x_2 - 1)^2 \quad (23)$$

$$\text{Subjected to } h(x) = x_2 - x_1^2 = 0 \quad (24)$$

$$\text{where } -1 \leq x_i \leq 1 \quad (i = 1, 2)$$

Variation of function value with respect to number of iterations for the method is given in Figure.1.

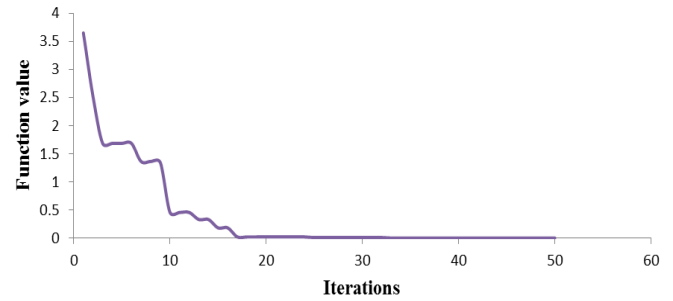


Figure.1. Variation of test function value w.r.t. no. of iterations

It is observed from the Figure.1 that the hybrid simplex method efficiently converges the standard test function towards the minimum function value.

6.2 ELD without Valve Point Loading

Three generators and six generators power system problems are undertaken to demonstrate the applicability of the method to optimize the generation schedule.

6.2.1 Three Generator Electrical Power System:

The optimal generation schedule has been calculated for loads 160 MW and 210 MW. The achieved generation schedule for prescribed loads by hybrid simplex method is given in Table 1 and Table 2.

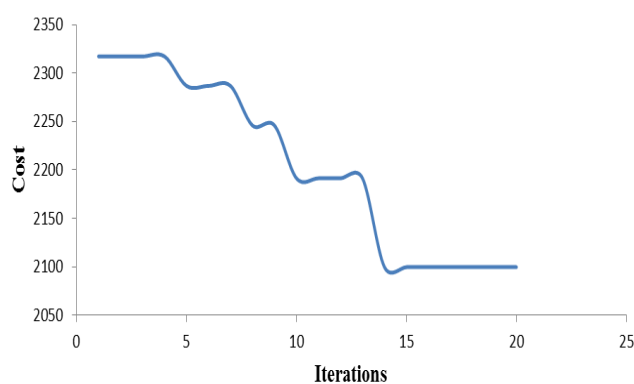
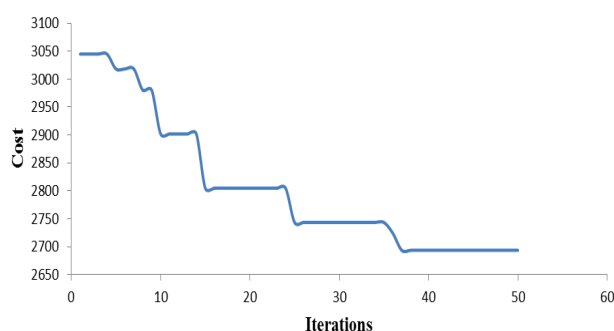
Table 1 Generation schedule for three generator system for load demand-160 MW

P_D (MW)	Iteration	P_{g1} (MW)	P_{g2} (MW)	P_{g3} (MW)	F (Rs/h)	P_L (MW)
160.0	1	42.1752	39.9572	100.9245	2317.2622	23.0569
	5	43.0944	34.5335	102.4144	2286.8259	20.0423
	8	34.5256	34.8515	102.0251	2245.9355	16.4022
	10	43.3086	25.5856	101.6573	2191.3743	10.5515
	14	40.0725	23.7406	97.8780	2099.5711	1.6911
	20	40.0725	23.7406	97.8780	2099.5711	1.6911

Table 2 Generation schedule for three generator system for load demand-210 MW

P_D (MW)	Iteration	P_{g1} (MW)	P_{g2} (MW)	P_{g3} (MW)	F (Rs/h)	P_L (MW)
210.0	1	65.5579	53.6372	133.1796	3044.9656	42.3744
	8	61.4912	48.4309	136.7951	2980.3484	36.7172
	10	64.3381	39.2366	135.1304	2901.9923	28.7051
	15	56.2628	36.0464	137.9470	2804.7021	20.2562
	25	50.1128	33.6542	141.2600	2743.4852	15.027
	37	47.4399	31.3345	141.6649	2693.6032	10.4393
	50	47.4399	31.3345	141.6649	2693.6032	10.4393

Variation in the operating cost has been depicted in Figure.2. and Figure.3.

**Figure.2.** Variation of cost w.r.t. number of iterations: Three generator with 160 load demand**Figure.3.** Variation of cost w.r.t. number of iterations: Three generator with 210 load demand

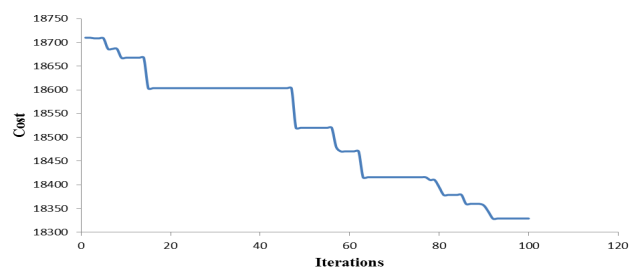
Comparison of results obtained by hybrid simplex method for three generator electrical power system for load demand of 160 MW and 210 MW with the results obtained by steepest descent method [12], conjugate gradient method [12] and Newton-Raphson method [8] is presented in Table 3. Hybrid simplex method gives better results with low transmission losses those depend upon schedule.

Table 3 Comparison of results for three generator system

Load Demand(MW)	Method	P_{g1} (MW)	P_{g2} (MW)	P_{g3} (MW)	F (Rs/h)	P_L (MW)
160	Steepest descent method	58.9287	62.5528	41.4465	2163.73	2.92800
	Conjugate gradient method	55.14402	55.12446	54.7345	2150.07	5.002980
	Newton Raphson method	57.5577	70.5238	37.9172	2176.023	5.998648
	Hybrid simplex method	40.0725	23.7406	97.8780	2099.5711	1.6911
210	Steepest descent method	78.1483	78.9839	62.5287	2747.11	9.66090
	Conjugate gradient method	73.99728	73.93189	72.93531	2716.88	10.86448
	Newton Raphson method	83.4010	95.6169	39.4862	2741.473	8.503935
	Hybrid simplex method	47.4399	31.3345	141.6649	2693.6032	10.4393

6.2.2 Six Generator Electrical Power System

The optimal generation schedule has been calculated for load 1800 MW. Variation in the operating cost with respect to iteration has been depicted in Figure.4.

**Figure.4.** Variation of function w.r.t. number of iterations: Six generator with 1800 load demand

Comparison of results obtained in this work by hybrid simplex method for load demand 1800MW for six generator power system with the results obtained by steepest descent method [12], conjugate gradient method [12] and newton raphson method [8] are presented in Table 4.

Table 4 Comparison of results for six generator system

Method	P_{g1} (MW)	P_{g2} (MW)	P_{g3} (MW)	P_{g4} (MW)	P_{g5} (MW)	P_{g6} (MW)	F (Rs/h)	P_L (MW)
Steepest descent method	268.23	276.53	502.77	373.47	306.33	196.47	18639.40	123.83
Conjugate gradient method	194.42	371.07	412.84	330.49	407.17	206.99	18523.80	123.02
Newton Raphson method	251.69	303.77	503.48	372.32	301.46	301.46	18721.39	130.15
Hybrid simplex method	228.04	281.41	489.87	354.51	349.44	185.99	18328.88	89.29

7. COMPARATIVE STUDY

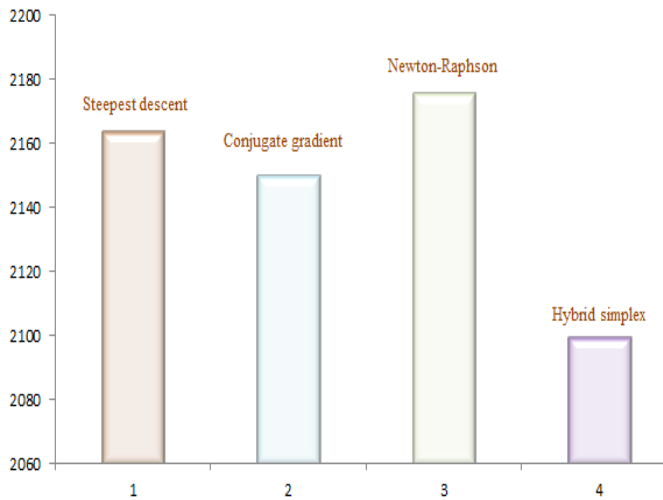


Figure.5. Operating cost comparison for load demand 160 MW

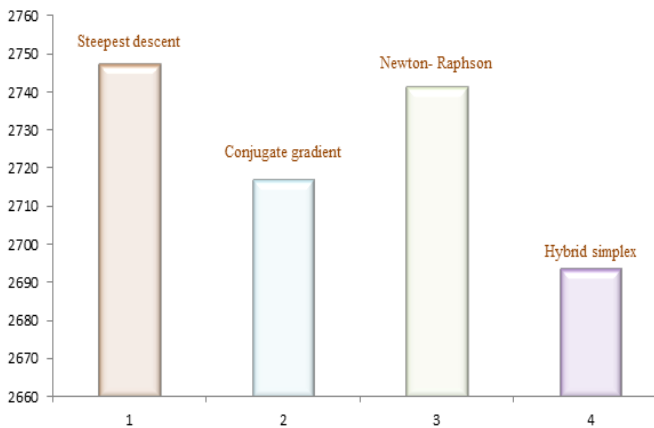


Figure.6. Operating cost comparison for load demand 210 MW

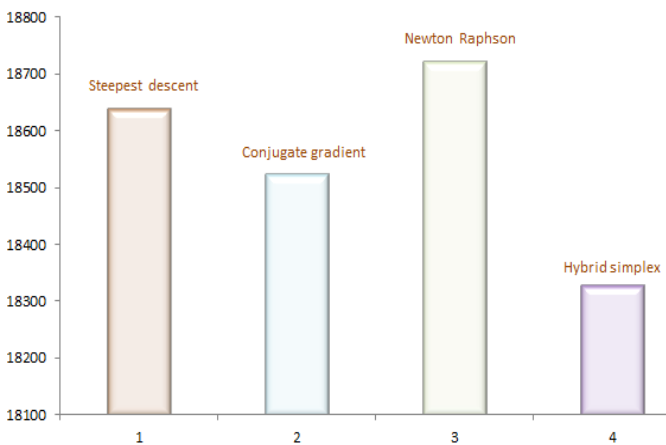


Figure.7. Operating cost comparison for load demand 1800 MW

Hybrid simplex method gives better results with lower transmission losses. The following figures depict the comparative operating costs of the steepest descent method, conjugate gradient method, newton raphson method and hybrid simplex method.

8. CONCLUSION

The effectiveness of the developed program is tested for test function and for different generator set systems i.e. for 3-generator and 6-generator electrical power system. The electrical power systems were taken considering valve point loading and without valve point loading. The total cost obtained from the hybrid simplex method is less as that of steepest descent method, conjugate gradient method and newton-raphson method. It is seen from Figure.5, Figure.6 and Figure.7 that hybrid simplex method provides more robust results as compared to the other three methods

9. ACKNOWLEDGEMENT

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